

Time Domain Full Waveform Inversion involving Discontinuous Galerkin approximation

Waves 2019

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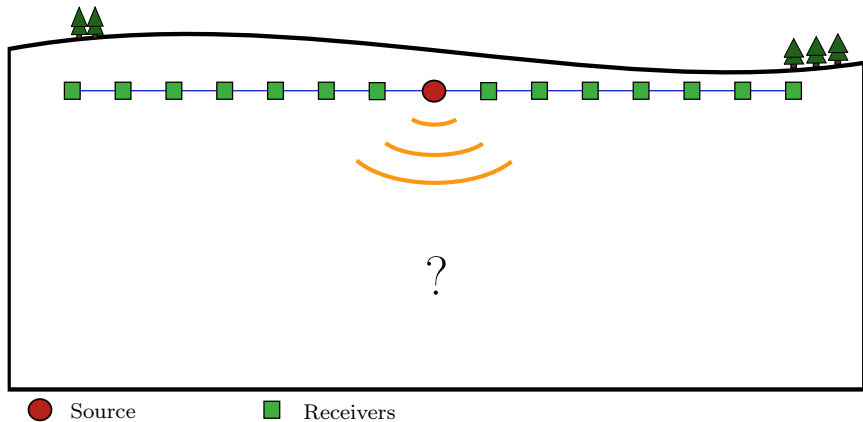
Second year PhD Student

Inria - Magique 3D - DIP

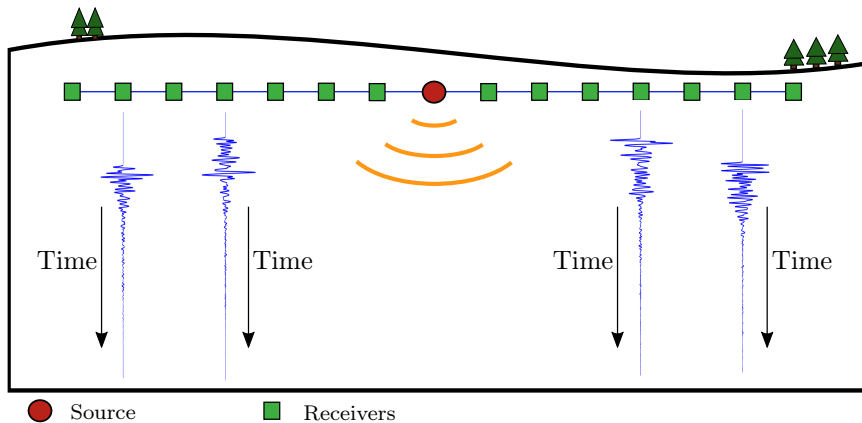
Pau, FRANCE



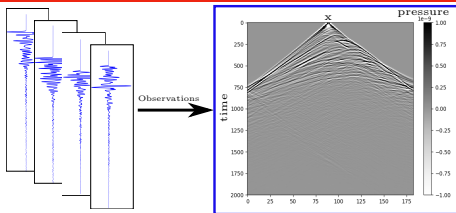
Seismic Acquisition



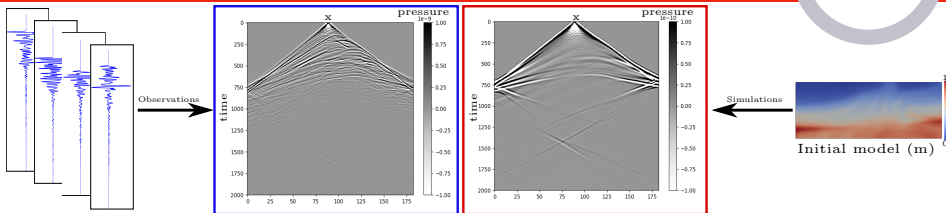
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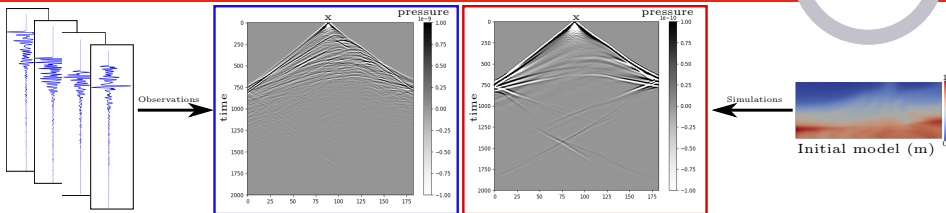
FWI Workflow



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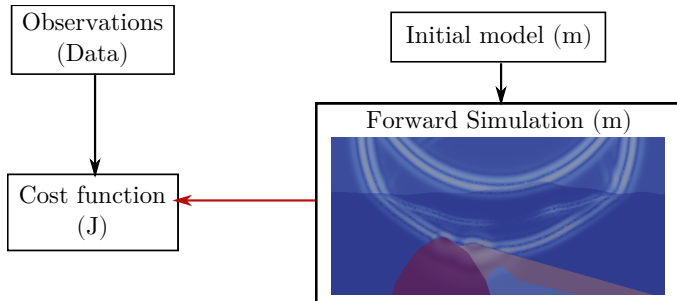
Cost function to minimize :

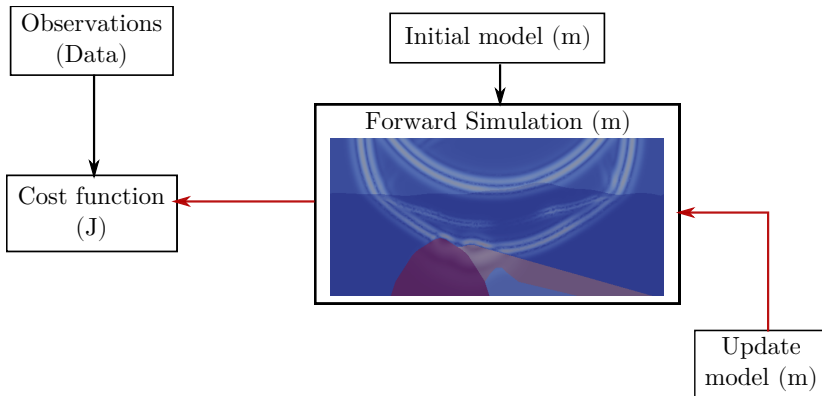
$$\mathcal{J}(\mathbf{m}) = \frac{1}{2} \|\mathbf{d}_{obs} - \mathcal{F}(\mathbf{m})\|^2$$

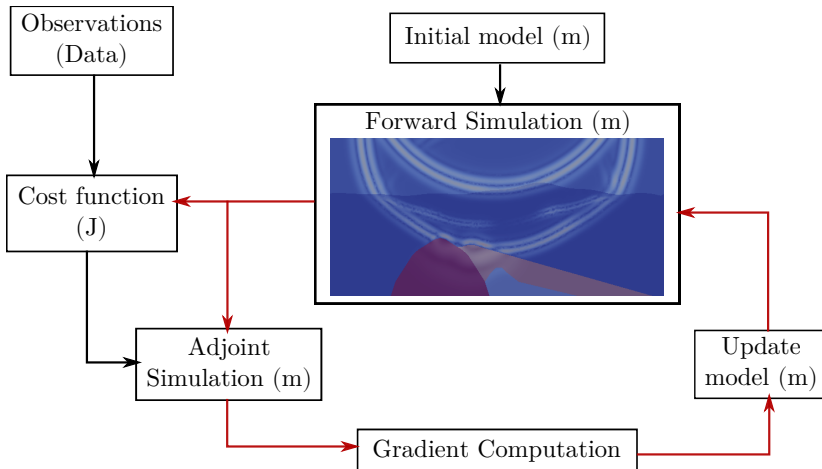
- ▶ $\mathcal{F}(\mathbf{m})$ is the restriction on the receivers of the simulated waves in the medium \mathbf{m} . (With $\mathbf{m} = \mathbf{c}, \rho, \kappa \dots$)
- ▶ FWI iterates until $\mathcal{J}(\mathbf{m}) \rightarrow 0$

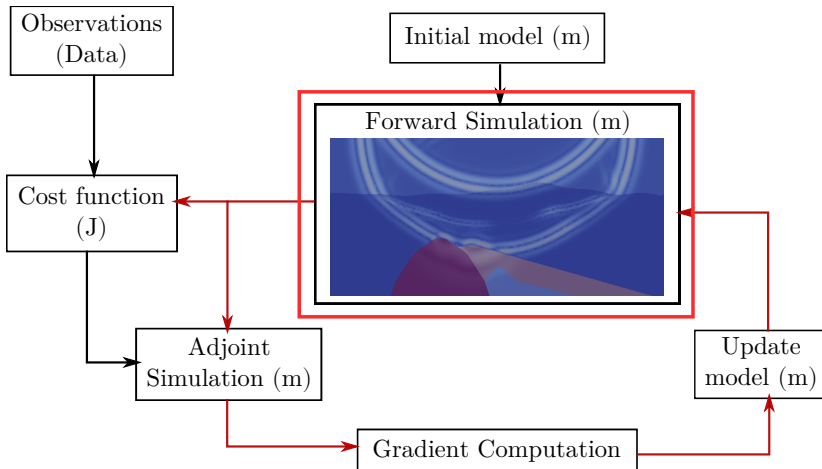
[1] Patrick Lailly
The seismic inverse problem as a sequence of before stack migrations
Conference on Inverse Scattering

[2] Albert Tarantola
Inversion of seismic reflection data in the acoustic approximation
Geophysics, Vol. 49, 1984



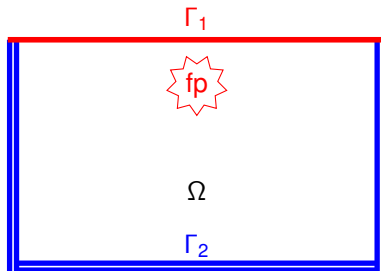






First order acoustic wave equation

$$\left\{ \begin{array}{l} \frac{1}{\rho \mathbf{c}^2} \frac{\partial \mathbf{p}}{\partial t} + \nabla \cdot \mathbf{v} = f_p \quad \text{on } \Omega \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{p} = 0 \quad \text{on } \Omega \\ \mathbf{p} = 0 \quad \text{on } \Gamma_1 \\ \frac{\partial \mathbf{p}}{\partial t} + \mathbf{c} \nabla \mathbf{p} \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_2 \\ \mathbf{p}(0) = 0, \quad \mathbf{v}(0) = 0 \end{array} \right.$$



Domain with Absorbing Boundary Conditions

Space Discretization : Discontinuous Galerkin Elements

- ▶ Nodal (Lagrangian / Jacobian)
- ▶ Modal (Bernstein-Bézier)

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Semi-discretized model :

$$\frac{\partial}{\partial t} \bar{\mathbf{U}}(t) = A \bar{\mathbf{U}}(t) + \bar{\mathbf{F}}(t)$$

with :

$$\bar{\mathbf{U}}(t) = \begin{pmatrix} \bar{\mathbf{P}}(t) \\ \bar{\mathbf{V}}(t) \end{pmatrix}$$

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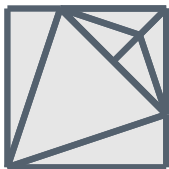
Time schemes :

- ▶ Runge Kutta 2/4
- ▶ Adams Bashforth 3

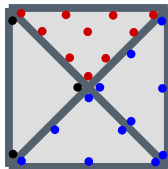


Assets of Discontinuous Galerkin Methods :

- ▶ Unstructured grid (enable to match the topography and medium irregularities)
- ▶ Robust to physical discontinuities
- ▶ hp-adaptivity
- ▶ Massively parallel performance properties



h-adaptivity



p-adaptivity with P1,
P2, P3 elements

Time Domain Full Waveform Inversion

- Seismic Acquisition

- FWI Workflow

- Forward Discretization

Adjoint Studies

- Adjoint then Discretized

- Discretize then Adjoint

Some Results

- 1D Preliminary tests

- 2D Time Domain FWI Results

- 2D Multiscale Reconstruction

Lagrangian functional [1] :

$$\mathcal{L}(\hat{\mathbf{u}}, \hat{\boldsymbol{\lambda}}, \mathbf{m}) = \frac{1}{2} \|\mathbf{d}_{obs} - \mathcal{R}(\hat{\mathbf{u}})\|^2 + \langle \text{Forward}_{\mathbf{m}}(\hat{\mathbf{u}}) - \mathbf{f}_p, \hat{\boldsymbol{\lambda}} \rangle$$

If $\hat{\mathbf{u}} = \mathbf{u}$ Solution of the Direct Problem $\iff (\text{Forward}_{\mathbf{m}}(\mathbf{u}) - \mathbf{f}_p = 0)$:

$$\mathcal{J}(\mathbf{m}) = \mathcal{L}(\mathbf{u}, \hat{\boldsymbol{\lambda}}, \mathbf{m})$$

[1] Plessix R-E

A review of the adjoint-state method for computing the gradient of a functional with geophysical applications
Geophysical Journal International, Volume 167, Issue 2, 2006

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$$(\mathcal{R}^* \mathbf{d}_{obs} - \mathbf{u}) + \text{Forward}_{\mathbf{m}}^*(\boldsymbol{\lambda}) = 0$$

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For $\text{Forward}_{\mathbf{m}}(\mathbf{u}) - f_p = 0$:

$$\partial_{\mathbf{m}_i} \mathcal{J}(\mathbf{m}) = \partial_{\mathbf{m}_i} \mathcal{L}(\mathbf{u}, \boldsymbol{\lambda}, \mathbf{m}) = \partial_{\mathbf{m}_i} \langle \text{Forward}_{\mathbf{m}}(\mathbf{u}), \boldsymbol{\lambda} \rangle$$

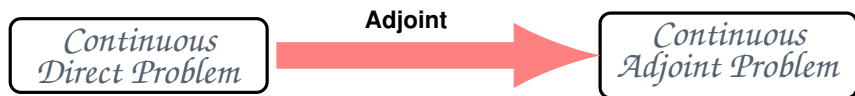
[1] Plessix R-E

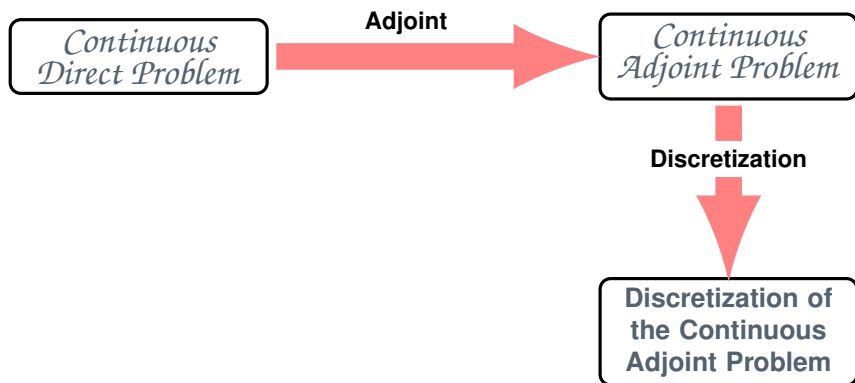
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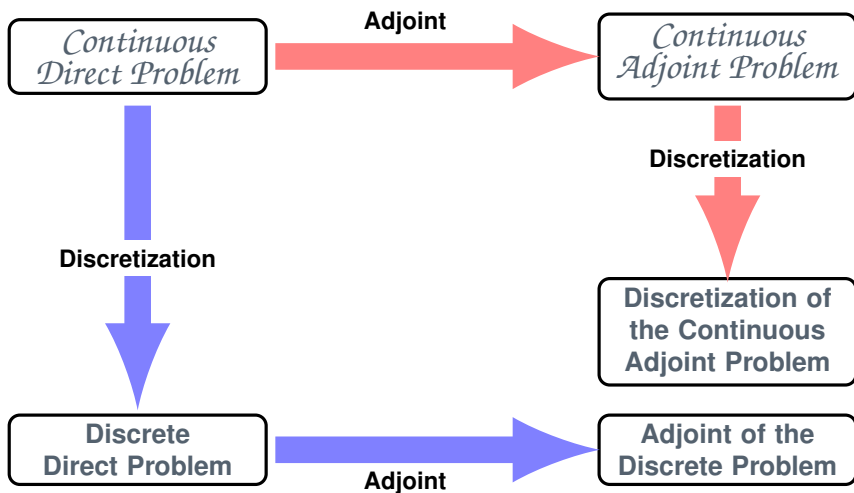
Adjoint Formulation

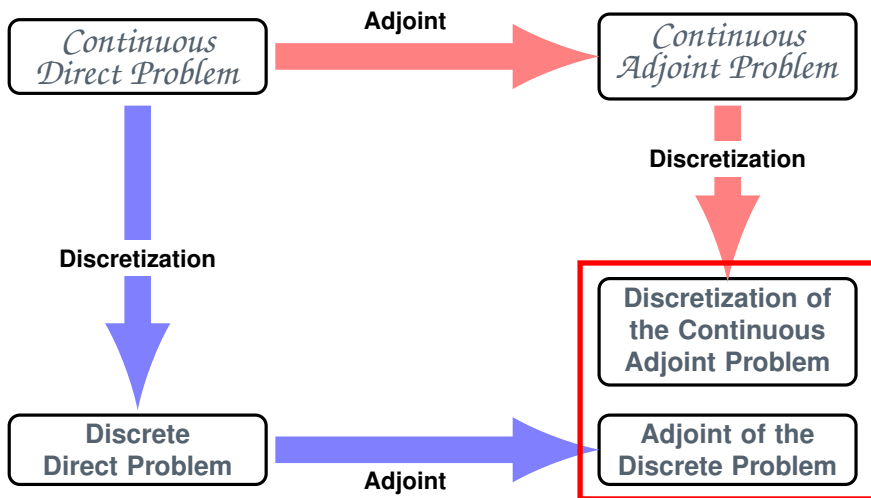


*Continuous
Direct Problem*









$$\mathcal{J}(\mathbf{p}) = \frac{1}{2} \| \mathbf{d}_{obs} - R\mathbf{p} \|^2$$

$$\left\{ \begin{array}{l} \frac{1}{\rho \mathbf{c}^2} \frac{\partial \mathbf{p}}{\partial t} + \nabla \cdot \mathbf{v} = f_p \quad \text{on } \Omega \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{p} = 0 \quad \text{on } \Omega \\ \mathbf{p} = 0 \quad \text{on } \Gamma_1 \\ \frac{\partial \mathbf{p}}{\partial t} + \mathbf{c} \nabla \mathbf{p} \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_2 \\ \mathbf{p}(0) = 0, \quad \mathbf{v}(0) = 0 \end{array} \right.$$

$$t \in [0, T]$$

$$\left\{ \begin{array}{l} \frac{1}{\rho \mathbf{c}^2} \frac{\partial \lambda_1}{\partial t} + \nabla \cdot \lambda_2 = \frac{\partial \mathcal{J}}{\partial \mathbf{p}} \quad \text{on } \Omega \\ \rho \frac{\partial \lambda_2}{\partial t} + \nabla \lambda_1 = 0 \quad \text{on } \Omega \\ \lambda_1 = 0 \quad \text{on } \Gamma_1 \\ \frac{\partial \lambda_1}{\partial t} - \mathbf{c} \nabla \lambda_1 \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_2 \\ \lambda_1(T) = 0, \quad \lambda_2(T) = 0 \end{array} \right.$$

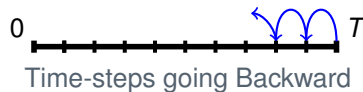
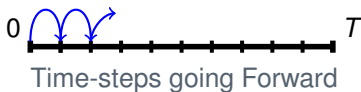
$$t \in [T, 0]$$

AtD : Adjoint then Discretized Strategy

$$\mathcal{J}(\mathbf{p}) = \frac{1}{2} \|\mathbf{d}_{obs} - R\mathbf{p}\|^2$$

$$\begin{cases} \frac{\partial \bar{\mathbf{U}}^n}{\partial t} = A\bar{\mathbf{U}}^n + \bar{\mathbf{F}}^n \\ \text{With : } \bar{\mathbf{U}}^n = \begin{pmatrix} \bar{\mathbf{P}}^n \\ \bar{\mathbf{V}}^n \end{pmatrix} \end{cases}$$

$$\begin{cases} \frac{\partial \bar{\boldsymbol{\Lambda}}^n}{\partial t} = A\bar{\boldsymbol{\Lambda}}^n + R^*(R\bar{\mathbf{U}}^n - \mathbf{d}_{obs}) \\ \text{With : } \bar{\boldsymbol{\Lambda}}^n = \begin{pmatrix} \bar{\boldsymbol{\Lambda}}_1^n \\ \bar{\boldsymbol{\Lambda}}_2^n \end{pmatrix} \end{cases}$$



DtA : Discretize then Adjoint Strategy

Example With RK4



All time scheme can be summed-up such as :

$$L\bar{U} = E\bar{F}$$

RK4 time-scheme leads to :

$$\bar{U}^{n+1} = B\bar{U}^n + C_0\bar{F}^n + C_{\frac{1}{2}}\bar{F}^{n+\frac{1}{2}} + C_1\bar{F}^{n+1}$$

$$L\bar{U} = E\bar{F} = \bar{G}$$
$$\begin{pmatrix} I & & & & \\ -B & I & & & \\ & -B & I & & \\ & & \ddots & \ddots & \\ & & & -B & I \end{pmatrix} \begin{pmatrix} \bar{U}^0 \\ \bar{U}^1 \\ \bar{U}^2 \\ \vdots \\ \bar{U}^n \end{pmatrix} = \begin{pmatrix} \bar{G}^0 \\ \bar{G}^1 \\ \bar{G}^2 \\ \vdots \\ \bar{G}^n \end{pmatrix}$$

All time scheme can be summed-up such as :

$$\mathbf{L}\bar{\mathbf{U}} = \mathbf{E}\bar{\mathbf{F}}$$

We are looking for a Discrete Adjoint state satisfying :

$$\mathbf{L}^*\bar{\mathbf{\Lambda}} = -\mathbf{R}^*(d_{obs} - \mathbf{R}\bar{\mathbf{U}})$$

With the adjoint operator \mathbf{L}^* satisfying :

$$\langle \mathbf{L}\bar{\mathbf{U}}, \bar{\mathbf{\Lambda}} \rangle = \langle \bar{\mathbf{U}}, \mathbf{L}^*\bar{\mathbf{\Lambda}} \rangle$$

All time scheme can be summed-up such as :

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With the adjoint operator \mathbf{L}^* satisfying :

$$\langle \mathbf{L}\bar{\mathbf{U}}, \bar{\boldsymbol{\lambda}} \rangle = \langle \bar{\mathbf{U}}, \mathbf{L}^*\bar{\boldsymbol{\lambda}} \rangle$$

$$\langle \bar{\mathbf{G}}, \bar{\boldsymbol{\lambda}} \rangle = \langle \bar{\mathbf{U}}, \bar{\mathbf{D}} \rangle \quad (\text{Adjoint Test})$$

Adjoint test succeeds \iff operator \mathbf{L}^* well established

DtA : Discretize then Adjoint Strategy

Example with RK4



RK4 time-scheme leads to :

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$$\mathbf{L}\bar{\mathbf{U}} = \mathbf{E}\bar{\mathbf{F}} = \bar{\mathbf{G}}$$
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So :

$$\mathbf{L}^* = \begin{pmatrix} I & -B^* & & & \\ & I & -B^* & & \\ & & \ddots & \ddots & \\ & & & I & -B^* \\ & & & & I \end{pmatrix}$$

Adjoint Then Discretize

- + Physical approach
- + Same discrete operators for Forward and Backward
- - Approximate gradient [1]
- Consistent with the discretization

Discretize then Adjoint

- + Numerical approach
- + Has an Adjoint Test
- Tremendous work to develop the adjoint operators
- Non-consistency of the adjoint state [2]

[1] Sirkes, Ziv and Tziperman, Eli
Finite Difference of Adjoint or Adjoint of Finite Difference ?
1997

[2] Sei Alain and Symes William
A Note on Consistency and Adjointness for Numerical Schemes
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- Seismic Acquisition

- FWI Workflow

- Forward Discretization

Adjoint Studies

- Adjoint then Discretized

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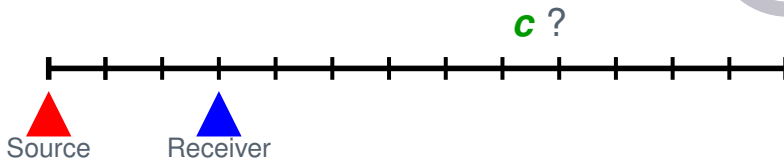
Some Results

- 1D Preliminary tests

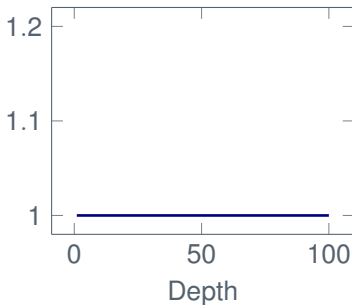
- 2D Time Domain FWI Results

- 2D Multiscale Reconstruction

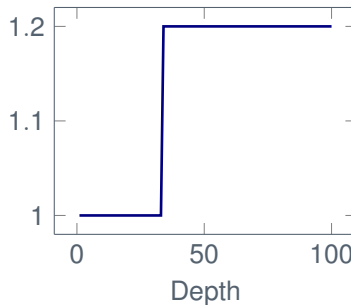
1D Preliminary tests



Initial **c** Model



Target **c** Model



1D Preliminary tests :



1D FWI :

- ▶ Lagrange / B-Bézier Operators
- ▶ RK4 / AB3 time-schemes

Gradient expression :

$$\nabla_{\mathbf{c}} \mathcal{J} = - \int_0^T \int_{\Omega} \frac{2}{\rho \mathbf{c}^3} \frac{\partial \mathbf{p}}{\partial t} \lambda_1 d\Omega dt$$

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Adjoint test passed with :

- ▶ With a canonical space inner-product
($\langle u, v \rangle_X = \sum_i u_i v_i$)
- ▶ With a M-space inner product
($\langle u, v \rangle_X^M = \langle Mu, v \rangle_X$)

```
./run
```

```
--- Adjoint test ----
```

```
inner product U/D 553123.57586755091
```

```
inner product G/Q 553123.57586756046
```

1D FWI :

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- ▶ With a canonical space inner-product
 $(\langle u, v \rangle_X = \sum_i u_i v_i)$
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```
./run
```

```
--- Adjoint test ----
```

```
inner product U/D 553123.57586755091
```

```
inner product G/Q 553123.57586756046
```

```
./run
```

```
--- Adjoint test ----
```

```
inner product U/D -75077.332007383695
```

```
inner product G/Q -75077.332007386358
```

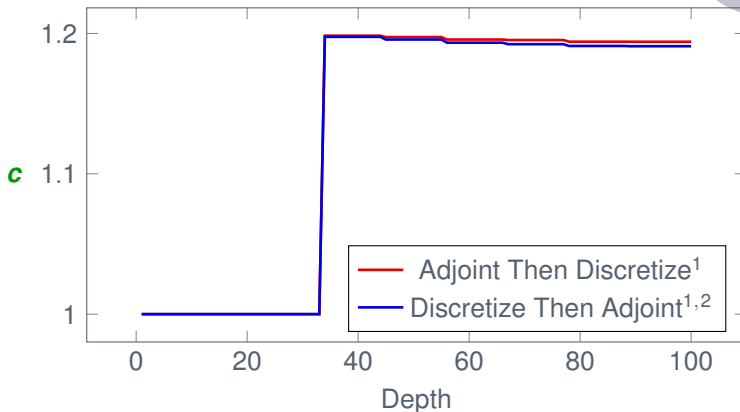
```
./run
```

```
--- Adjoint test ----
```

```
inner product U/D 125669.89223600870
```

```
inner product G/Q 125669.89223600952
```

1D Velocity Model Reconstructions



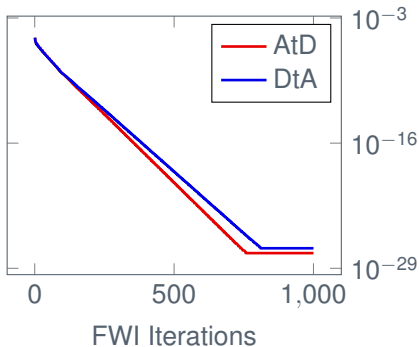
c Model at the 100th FWI iteration

¹With Bernstein-Bézier elements and AB3 time scheme

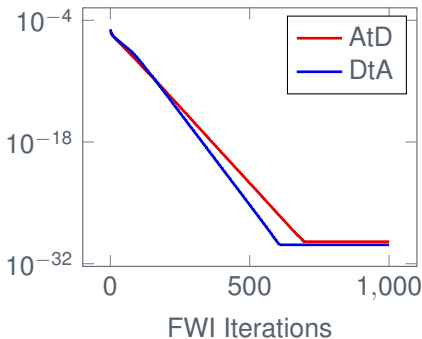
²With canonical scalar product

1D Velocity Model Reconstructions

With RK4 :



With AB3 :

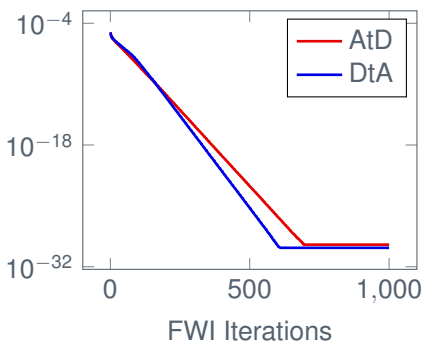


1D Velocity Model Reconstructions

With RK4 :



With AB3 :



- ▶ For RK4 scheme : AtD is slightly better than DtA
- ▶ For AB3 scheme : DtA is slightly better than AtD
- ▶ No predominant behaviour

2D FWI :

- ▶ Developed in Total environnement (DIP³)
- ▶ Nodal Space Operators (Lagrangian/Jacobian)
- ▶ Modal Space Operators (Bernstein-Bézier)
- ▶ Runge Kutta 2/4 and Adams Bashforth 3 time-schemes

Discretize Then Adjoint strategy not implemented :

- ▶ Tremendous task in a complex industrial code

³<http://dip.inria.fr/>

2D FWI :

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- ▶ Modal Space Operators (Bernstein-Bézier)
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Gradient expression :

$$\nabla_{\frac{1}{\kappa}} \mathcal{J} = \int_0^T \int_{\Omega} \frac{\partial \mathbf{p}}{\partial t} \lambda_1 d\Omega dt \quad \text{with : } \kappa = \rho \mathbf{c}^2$$

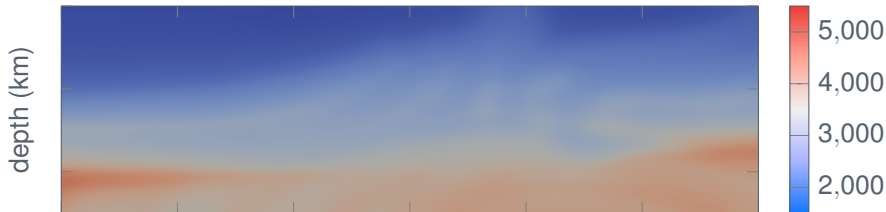
\mathbf{c} , ρ and κ Constant per elements

³<http://dip.inria.fr/>

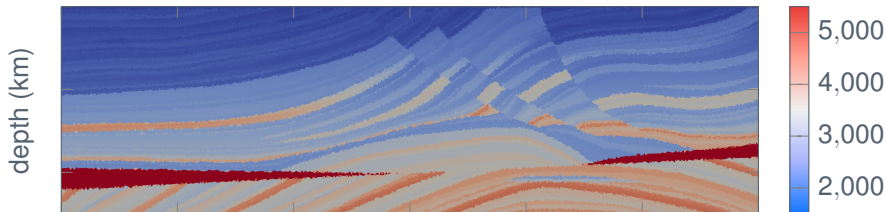
2D Time Domain FWI Reconstructions

Time-schemes comparison

Initial **c** Model



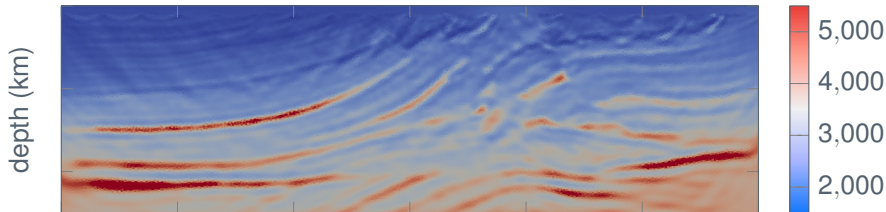
Target **c** Model



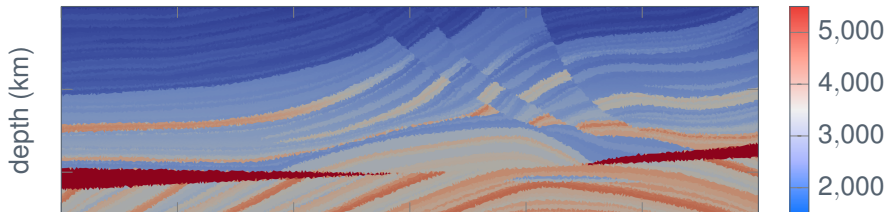
2D Time Domain FWI Reconstructions

Time-schemes comparison

RK2 Reconstructed **c** Model (30 iterations)



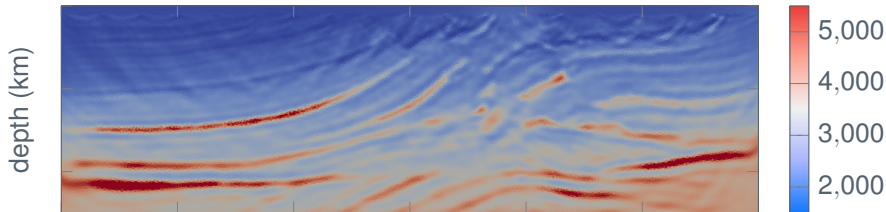
Target **c** Model



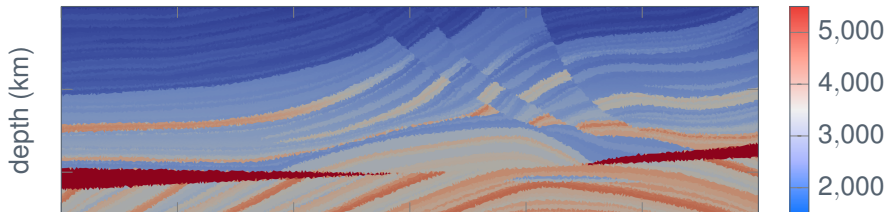
2D Time Domain FWI Reconstructions

Time-schemes comparison

RK4 Reconstructed **c** Model (30 iterations)



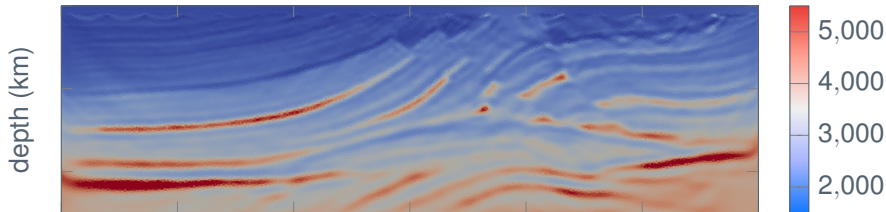
Target **c** Model



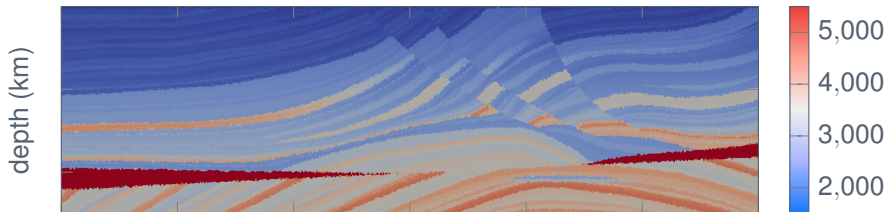
2D Time Domain FWI Reconstructions

Time-schemes comparison

AB3 Reconstructed **c** Model (30 iterations)



Target **c** Model



2D Time Domain FWI Reconstructions

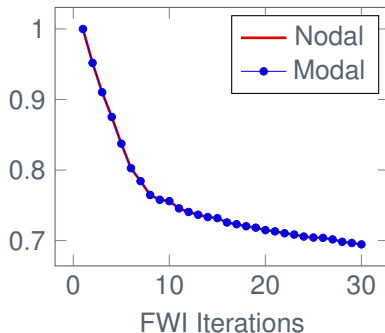
Nodal/Modal Comparison



24

- ▶ 47k P1 elements
- ▶ Time Scheme : AB3
- ▶ Constant ρ model ($\rho = 1$)
- ▶ 19 sources / 181 Receivers
- ▶ Noise : SNR=10
- ▶ 30 iterations
- ▶ 120 cores
- ▶ Nodal computation time : 5h10
- ▶ Modal computation time : 7h10^[1]

Cost function evolution :

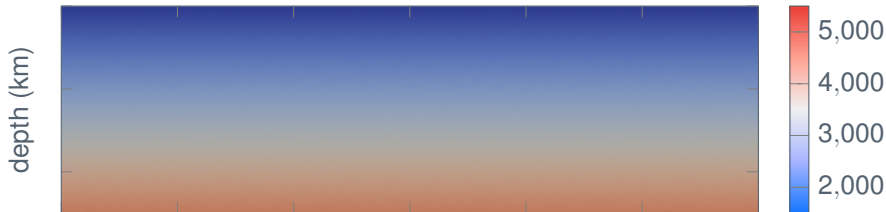


[1] Chan J. and Warburton T.
GPU-Accelerated Bernstein Bézier Discontinuous Galerkin Methods for Wave Problems
SIAM Journal on Scientific Computing 2017

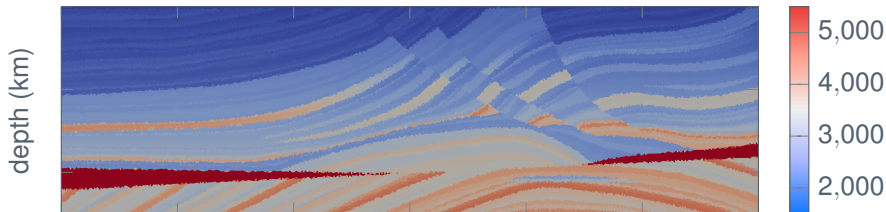
2D Multiscale Reconstructions

Reconstruction with an initial smooth model

Initial **c** Model



Target **c** Model

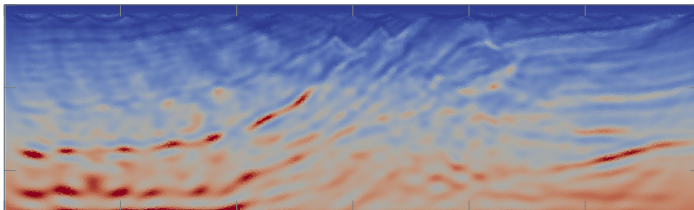


2D Multiscale Reconstructions

Reconstruction with an initial smooth model

Reconstructed model **c** Model (30 iterations AB3)

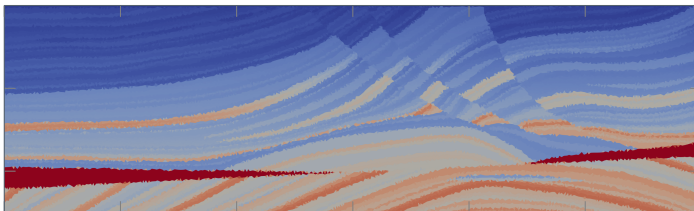
depth (km)



Target **c** Model

$m \cdot s^{-1}$

depth (km)



2D Multiscale Reconstructions

Multiscale Principle [1]

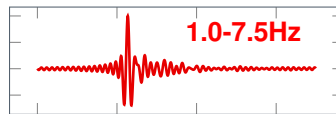
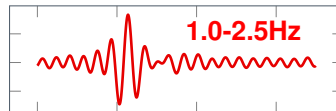
Filtered Traces :

p

Low Frequencies



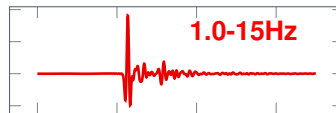
Reconstruct **coarse** structures



High Frequencies



Reconstruct **small** structures



[1] C. Bunks, F. M. Saleck, S. Zaleski, and G. Chavent
Multiscale seismic waveform inversion
GEOPHYSICS, Vol. 60, No. 5, 1995

2D Multiscale Reconstructions

Multiscale Principle [1]



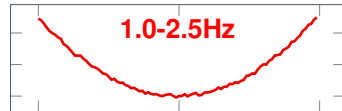
Heuristic Illustration :

\mathcal{I}

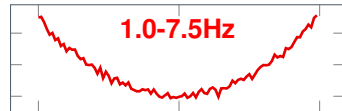
Low Frequencies



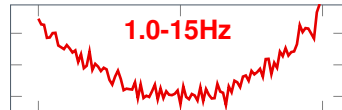
Reconstruct **coarse** structures



m



m



m

High Frequencies



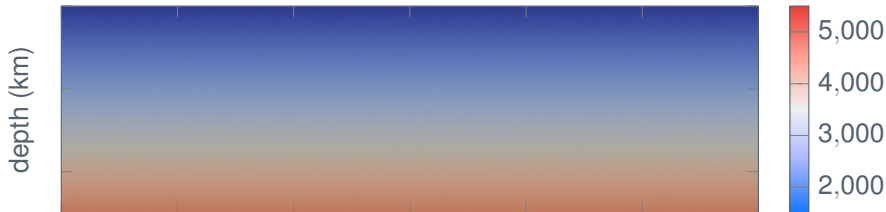
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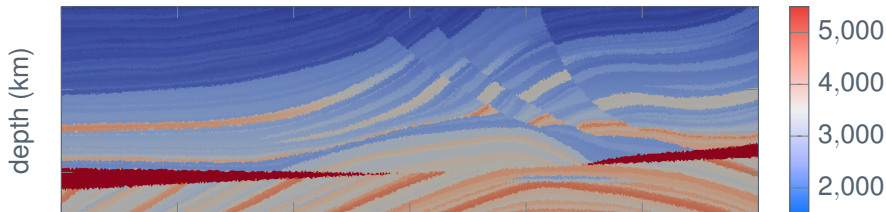
2D Multiscale Reconstructions

Reconstruction with an initial smooth model

Initial **c** Model



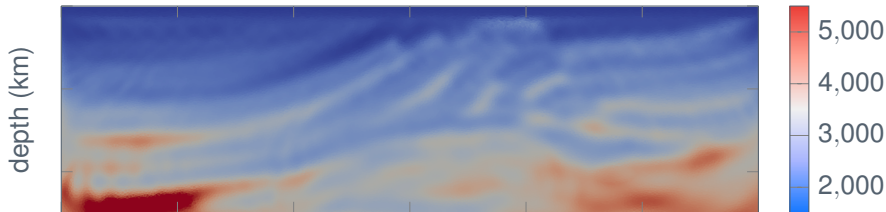
Target **c** Model



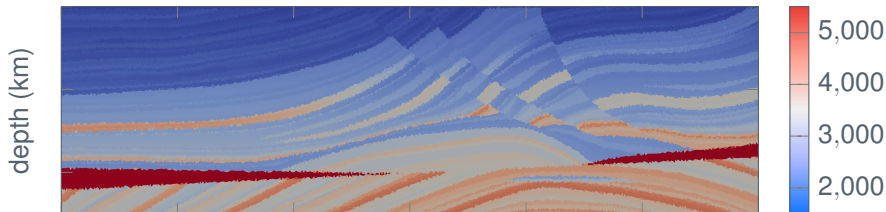
2D Multiscale Reconstructions

Reconstruction with an initial smooth model

Reconstructed **c** Model with 1.0-2.5Hz filter



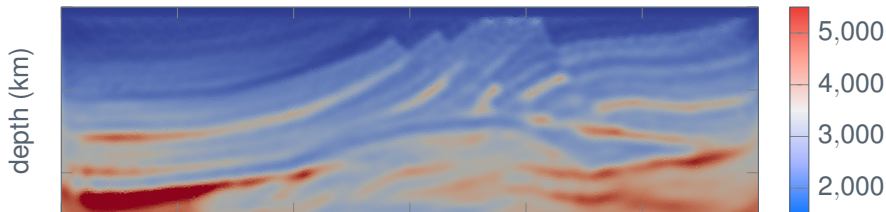
Target **c** Model



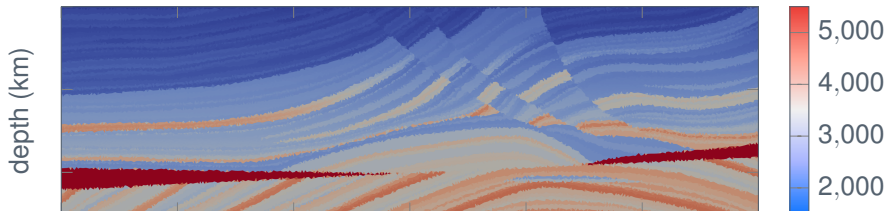
2D Multiscale Reconstructions

Reconstruction with an initial smooth model

Reconstructed **c** Model with 1.0-7.5Hz filter



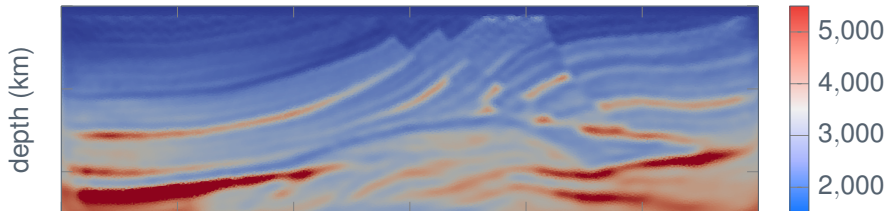
Target **c** Model



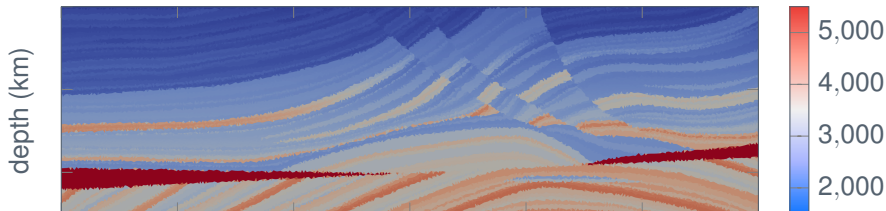
2D Multiscale Reconstructions

Reconstruction with an initial smooth model

Reconstructed **c** Model with 1.0-10Hz filter



Target **c** Model

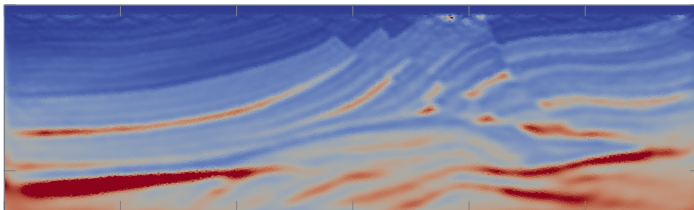


2D Multiscale Reconstructions

Reconstruction with an initial smooth model

Reconstructed **c** Model with 1.0-15Hz filter

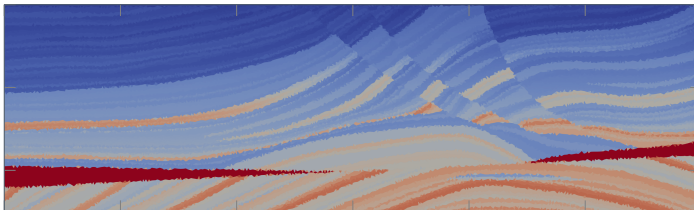
depth (km)



Target **c** Model

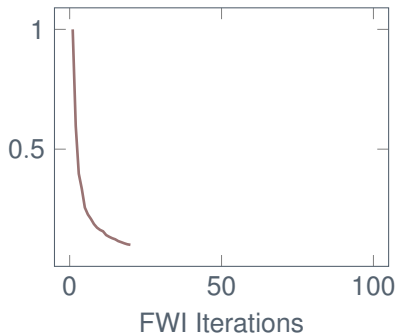
$m \cdot s^{-1}$

depth (km)



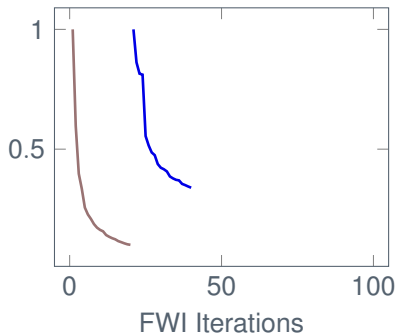
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- ▶ Noise : SNR=10
- ▶ 120 cores
- ▶ Computation time : 17h
- ▶ Frequencies : 1-2.5Hz

Cost function evolution :



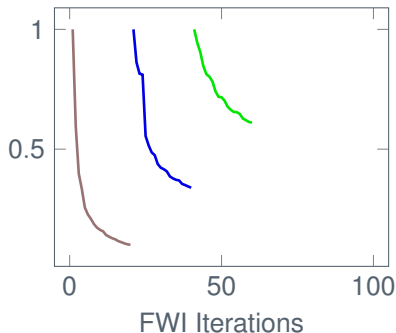
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1-5.0Hz

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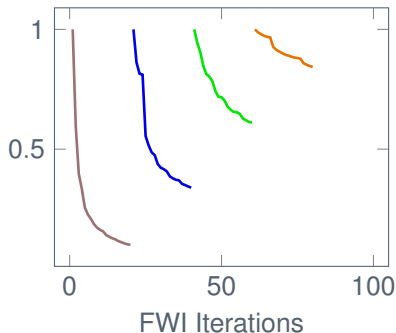
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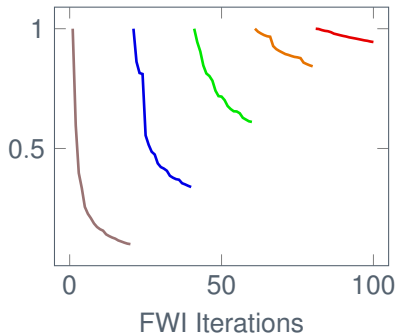
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- ▶ Computation time : 17h
- ▶ Frequencies : 1-2.5Hz,
1-5.0Hz, 1-7.5Hz, 1-10Hz,
1-15Hz

Cost function evolution :



Main Results :

- ▶ Comparison between adjoint formulations (**AtD** and **DtA**)
- ▶ 2D Acoustic Reconstruction performed with different discretization
- ▶ Multiscale FWI implemented and working on Marmousi

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Thank you.